EFFICIENT PRECISE ORBIT DETERMINATION OF LEO SATELLITES USING GPS

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ABSTRACT

Efficient precise orbit determination of LEO satellites plays an important role for near real-time studies of GPS satellite occultations for meteorological purposes. Precise point positioning for each epoch is one approach to achieve this goal. Using IGS orbits and precise clocks for the GPS satellites the positions are generated by the combination of code derived positions and phase derived position differences. Fitting an orbit based on a physical model to the positions promises to complement a procedure that meets the requirements regarding precision and processing speed. This efficient procedure is tested with data of TOPEX/POSEIDON.

INTRODUCTION

GPS receivers were used for spaceborne applications since the early 1980's. The development has been rather slow, however. The first missions where GPS was successfully used for orbit determination were the remote sensing satellite LANDSAT 5 launched in March 1984, the radar altimeter satellite TOPEX/POSEIDON launched in August 1992, and the atmosphere sounding satellite GPS/MET launched in April 1995. A number of launches of Low Earth Orbiters (LEOs) using GPS among other tracking data for precise orbit determination (POD) are planned in the next few months and years. The altitude range of those planned missions varies between 350 and 1000 km. Table 1 lists some information about missions scheduled for launch in the near future or which are considered for implementation (COSMIC, ACE, Metop).

Precise orbit determination of a LEO using GPS is not a new issue. A number of studies were already carried out. A recent overview may be found in Bisnath and Langley (1999). A new requirement, mainly driven by the meteorological community interested in data from atmospheric sounding satellites, is the availability of precise orbits in near real-time. For the processing of the GPS occultation data the orbit of the LEO needs an accuracy of the order of 0.1 mm/s in velocity. In this paper we will develop an

Table 1. LEO Missions Using Spaceborne GPS Receivers for Precise Orbit Determination (POD)

Satellite	Launch	Purpose
CHAMP	July 15, 2000	Gravity- and geomagnetic field, atmospheric limb sounding
SAC-C	November 21, 2000	Gravity- and geomagnetic field, atmosphere, ecosystems
JASON-1	August 10, 2001	Altimetry, oceanography (follow-on mission of TOPEX/POSEIDON)
GRACE	November 2001	Gravity field, climate experiment (two spacecraft)
GOCE	2005	Gravity field
COSMIC	2003	Atmospheric limb sounding
ACE	2004	Atmospheric limb sounding
Metop	2005	Atmospheric limb sounding

approach for LEO POD which is not based on the common double-differencing of the GPS observations but which uses zero-difference observations (code) and their epoch-by-epoch differences (phase). Tracking data from ground stations is needed only for the generation of the GPS clock corrections, a procedure which is completely independent from the LEO POD processing. The algorithms are simple and fast and thus the new approach promises to meet the requirements for both, processing speed and accuracy, for the GPS occultation missions. In the first section we shortly describe the algorithms, in the second section we present results and validations.

THE ALGORITHMS

GPS Precise Satellite Clocks

If other than double-differences between the GPS observations are formed, the clock corrections from both, the GPS satellites and the LEO GPS receiver are not removed by the processing procedure. The corrections for the LEO clocks are determined together with the positions whereas those for the GPS satellites are computed in a seperate step. An efficient algorithm to generate precise clock values for the GPS satellites at each observation epoch of a permanent GPS network may be found in Bock et al. (2000). In a first step code and phase observations from a worldwide tracking network are analyzed together in order to establish time series of precise satellite clock estimates for the entire GPS constellation. Orbits, troposphere parameters, and station coordinates are constrained to the values taken from the IGS (International GPS Service) or from one of its analysis centers. Using the ionosphere-free linear combination results in a simple observation equation to estimate satellite and receiver clocks. To eliminate the initial ambiguity term in the phase processing the observation differences between subsequent epochs are formed and the clock differences estimated. For both, code and phase, an iterative process is set up to check for data problems and cycle slips. In the second step the two clock sets, the code-derived absolute clock corrections and the phase derived clock difference corrections, are combined into one consistent time series of clock corrections. The matrix of the normal equation system associated with these corrections has a (symmetric) tridiagonal structure which allows to solve the normal equation system with a very efficient algorithm. This procedure is a simple and quick way to generate high rate clocks both for the GPS satellites and for the ground-based receivers. The precise clocks of the GPS satellites are used in the subsequent step for the LEO POD.

LEO Point Positioning

The processing of LEO GPS data is similar to the GPS clock processing procedure outlined in the previous section. We use code observations for each epoch to determine positions, and phase difference observations of each epoch difference to determine position differences of the LEO. The algorithm is somewhat more involved because the dependence of the observables on the coordinates is not linear. Let us have a look at the code processing first. We use the ionosphere-free linear combination of the P1- and P2-code measurements. The orbits of the GPS satellites are known from IGS and the GPS clock corrections are taken from the clock generation procedure described in the previous section. With these assumptions the observation equation of one code observation from the LEO to one GPS satellite at epoch t_i reads as follows:

$$p = |\mathbf{r_{GPS}} - \mathbf{r_L}| + c \cdot \Delta t_L \qquad (1)$$

with the ionosphere-free linear combination of the observed code pseudorange p, the position vector $\mathbf{r_{GPS}}$ of the observed GPS satellite at epoch $t_i - \tau$, where τ is the signal travel time, the unknown position vector $\mathbf{r_L}$ of the LEO, the speed of light c, and the unknown LEO clock correction Δt_L.

We may write Eq. (1) for each observation acquired at one epoch. If more than four satellites are in view, we may estimate the coordinates x_L, y_L, z_L and the clock correction Δt_L for the LEO using a least squares adjustement for each epoch. The results are LEO point positions and clock corrections with the accuracy allowed by code (0.5 - few metres).

For the processing of the phase observations we use the ionosphere-free linear combination of L1- and L2-measurements and form differences between subsequent epochs to eliminate the initial ambiguity term. As a consequence we may estimate only position differences between subsequent epochs. The observation equation for the phase difference between the epochs t_i and t_{i+1} for one GPS satellite reads as (signal travel time corrections omitted for simplicity):

$$\begin{aligned}\Delta\phi(t_{i,i+1}) &= |\mathbf{r_{GPS}}(t_{i+1}) - \mathbf{r_L}(t_{i+1})| + c\cdot\Delta t_L(t_{i+1}) - (|\mathbf{r_{GPS}}(t_i) - \mathbf{r_L}(t_i)| + c\cdot\Delta t_L(t_i)) \\ &= |\mathbf{r_{GPS}}(t_{i+1}) - \mathbf{r_L}(t_i) - \Delta\mathbf{r_L}(t_{i,i+1})| - |\mathbf{r_{GPS}}(t_i) - \mathbf{r_L}(t_i)| + c\cdot\Delta\Delta t_L(t_{i,i+1}) \qquad (2)\end{aligned}$$

with $\Delta\mathbf{r_L}(t_{i,i+1}) = \mathbf{r_L}(t_{i+1}) - \mathbf{r_L}(t_i)$ and $\Delta\Delta t_L(t_{i,i+1}) = \Delta t_L(t_{i+1}) - \Delta t_L(t_i)$. Formally, the unknown coordinates $\mathbf{r_L}(t_i)$ for the first of the two epochs remain in the equation as parameter. We may see, however, that the coefficient of this parameter in the linearized observation equation is several orders of magnitude smaller than that for $\Delta\mathbf{r_L}$ and is proportional to the time interval $t_{i+1} - t_i$. If an a priori orbit of good quality is available and if the sampling rate is high enough (e.g., 30 seconds) we may neglect the correction term for $\mathbf{r_L}(t_i)$. As a priori orbit we may use the code positions determined in the previous step or an orbit prediction from the processing of the previous arc. Exactly as in the case of the GPS clock corrections we may combine the clock corrections Δt_L generated from code with the clock correction differences $\Delta\Delta t_L$ generated from phase differences to satellite clock corrections with an accuracy given by the phase.

Orbit Determination

The positions estimated from code observations and position differences estimated from phase observations may eventually be used as "pseudo-observations" to fit an orbit based on the physical model using a least squares adjustement. Important perturbing forces are due to the Earth potential, atmospheric drag, and solar radiation pressure. For our tests we used Earth potential terms up to degree and order 70 either from GRIM5 or from EGM96 potential models. Atmospheric drag is computed using the density model of MSISEM90 (Hedin, 1991). Satellite specific information such as antenna offset and attitude motion has to be implemented for each LEO seperately.

RESULTS AND VALIDATION

Quality of GPS Satellite Clocks

High rate satellite clock corrections are generated automatically each day using the CODE (Center for Orbit Determination in Europe) rapid orbits and troposphere parameters for the IGS-stations. To check the quality of the clock corrections we compare them with the clock corrections delivered to IGS by JPL (Jet Propulsion Laboratory) every week. JPL so far is the only IGS analysis center providing 30-second clocks. The differences between the two solutions are shown in Figure 1 for two satellites (PRN 5 and 6) for June 10, 2000. The contributions of the reference clocks have been removed. The clock differences are well below one nanosecond. An independent quality check was performed by the IGS Analysis Center Coordinator (ACC) who reprocessed one week (GPS week 1064) of the IGS Rapid Combination with our new clock corrections included into the clock combination. In Table 2 the RMS values and weights of our clocks are summarized for this week. The weights indicate the relative contribution of the clock corrections to the clock combination with respect to the clock corrections delivered by other analysis centers. Normally,

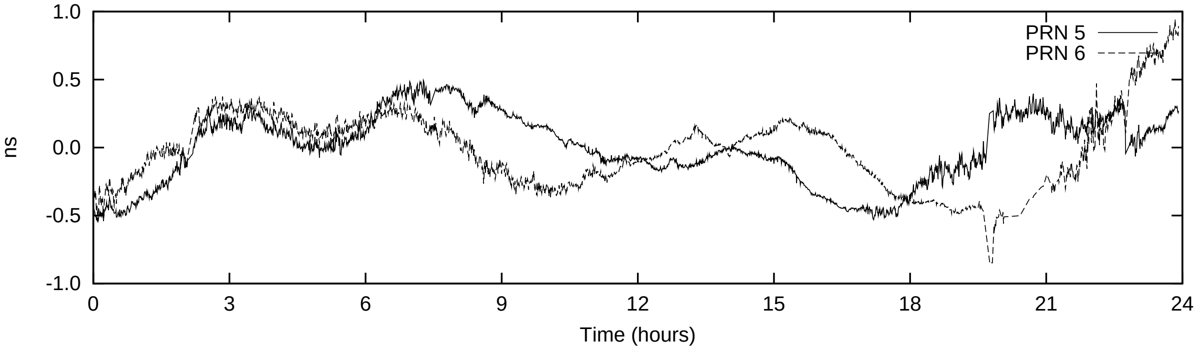


Fig. 1. Differences between JPL- and our clock corrections for PRN 5 and 6.

Table 2. RMS and Weight of our Clock Corrections in Reprocessed IGS Rapid Combination (GPS Week 1064)

Day	0	1	2	3	4	5	6
RMS (ns)	0.32	0.38	0.33	0.36	0.37	0.37	0.40
Weight(%)	13	9	19	22	9	14	14

four to six analysis centers contribute to the clock combination. The weights in Table 2 thus indicate that our clocks would contribute in a significant way to the IGS combination if delivered routinely. Table 2 also demonstrates that using the simple algorithm outlined above, we are able to generate satellite clock corrections of comparable quality as those delivered by the IGS analysis centers.

Validation of LEO Orbits

The procedures of our new POD approach were tested using data from TOPEX/POSEIDON (days 51-54, 1997). For this time span we may compare our orbits with a reference orbit which stems from a combination of DORIS (Doppler Orbitography and Radiopositioning Integrated by Satellite)- and SLR (Satellite Laser Ranging)- observations and nominally has an accuracy of 3 cm (Tapley et al. (1994) and Nouël et al. (1994)) in radial direction. To check the quality of the positions and position differences determined by the algorithm described above they are compared with the corresponding position and position differences of the TOPEX/POSEIDON reference orbit.

In Figure 2 the comparison for the differences $(\mathbf{r_{i+1}} - \mathbf{r_i})_{new} - (\mathbf{r_{i+1}} - \mathbf{r_i})_{ref}$ are shown seperately for the radial, alongtrack, and crosstrack components. The RMS of the comparison is 2.5 cm for the radial component and 1 cm in the other two components. No strong systematic behaviour is evident in the residuals. For the comparison of the code-derived LEO positions the RMS is 3.1 m for the radial, 1 m for the alongtrack, and 1.3 m for the crosstrack component. The results show that the proposed point positioning algorithm reconstructs the positions and the position differences of TOPEX/POSEIDON with an accuracy as expected from the observables considered.

Influence of GPS Satellite Clocks and Orbits on LEO Orbit

The IGS provides several orbit products that may be used as input for our clock generation program. For the near real-time applications it is necessary to have precise orbits for the GPS satellites in near real-time, too. Best suited for this purpose are the IGS Ultra Rapid orbits which are an official IGS product since November 2000. The Ultra Rapid orbits are delivered twice a day at 3^h and 15^h UT. GPS data from the 40-50 IGS stations delivering hourly tracking data are used to generate the orbits. The average age of the 24 hour predictions is 9 hours. Currently the quality of the predictions is 35 cm (weighted RMS)

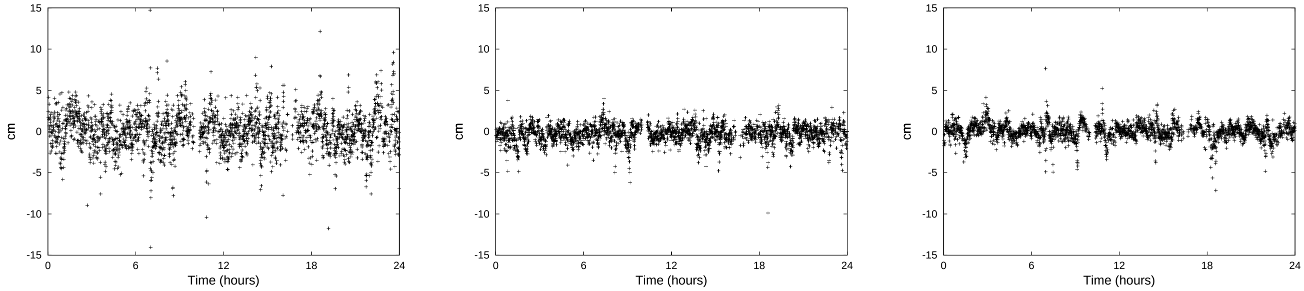
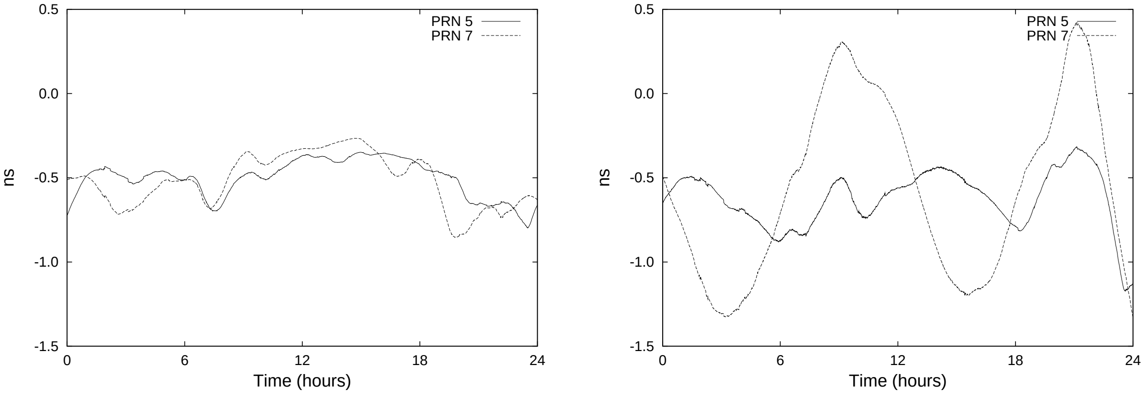


Fig. 2. Position differences in radial (left), alongtrack (middle) and crosstrack (right) direction compared to the TOPEX/POSEIDON reference orbit for day 54, 1997.



(a) Solution 2 - Solution 1

(b) Solution 3 - Solution 1

Fig. 3. Clock correction differences between different solutions for PRN 5 and 7, 10 June 2000.

compared to 5 cm for the IGS Rapid orbits. An additional problem for the clock estimation process is the lack of information about troposphere zenith delays for the tracking stations in near real-time. This is why a troposphere model has to be used. The impact of the use of predicted orbits and troposphere models on the quality of the satellite clock estimates and on the LEO POD process needs to be further studied. To estimate the impact three independent clock correction solutions are generated for one day (June 10, 2000) by using the following three options

1. CODE Rapid orbits + estimated troposphere zenith delays from CODE (reference solution)
2. CODE Rapid orbits + troposphere model
3. IGS Ultra Rapid orbits + troposphere model.

Solution 1 is taken as a reference solution. Figure 3(a) shows the differences between the clock solutions 2 and 1 for two arbitrarily selected satellites (PRN 5 and 7), Figure 3(b) those between the clock solutions 3 and 1. We observe that solution 2 (troposphere model) essentially shows an offset of about 0.5 ns with respect to solution 1 (troposphere parameters introduced). In addition to an offset, solution 3 (troposphere model and ultra rapid orbits) also shows a periodic variation with respect to solution 1, which is due to the different orbits used for the generation of the clock solutions.

The GPS clock and orbit errors introduced by using IGS Ultra Rapid orbits and a troposphere model influence the accuracy of the LEO orbit. Part of the biases introduced may be absorbed by the LEO clock corrections. In addition, orbital dynamics helps to reduce the effect systematic biases in the observables may have on the orbit. The longer the orbital arc, the more efficient this rejection of observation biases. The optimum arc length is, however, a trade-off between observation biases and biases due to orbit modeling errors. Figure 4 indicates that an arc length of two revolutions is long enough to reduce the effects due to the use of ultra rapid orbits and troposphere models to values below the orbit accuracy required for occultation studies.

CONCLUSIONS

We have presented a procedure for efficient LEO POD. GPS satellite clock corrections are determined in an independent step using code and phase observations from a fiducial network. In a following step the GPS clocks and orbits are introduced in the orbit determination procedure for the LEO. The proposed approach promises to meet the requirements of accuracy and processing speed for near real-time applications. The performance of the algorithm has to be confirmed using data from the upcoming LEO missions.

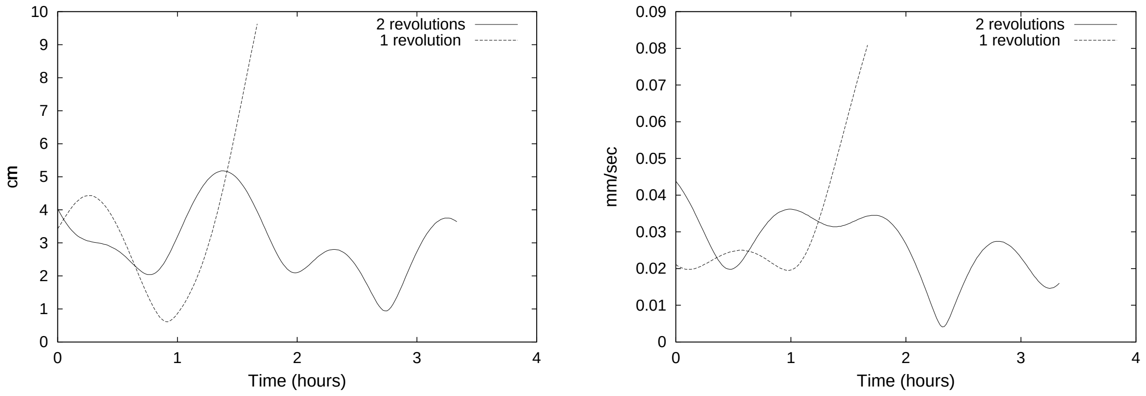


Fig. 4. Differences in LEO position (left) and velocity (right) due to GPS orbit and clock errors for one and two revolutions.

ACKNOWLEDGMENTS

The GPS data, the reference orbit and additional information of TOPEX/POSEIDON was made available by CNES (Centre National d'Etudes Spatiales), Toulouse, France. These contributions are gratefully acknowledged.

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